## Rutgers University: Algebra Written Qualifying Exam January 2017: Problem 4

Exercise. Prove that $D_{6}$ and $A_{4}$ are not isomorphic. (Here $D_{6}$ e is the symmetry group of the hexagon and $A_{4}$ is the alternating group of even permutations on 4 letters.)

## Solution.

$$
\begin{aligned}
& A_{4}=\{(1), \\
& \left(\begin{array}{ll}
1 & 2
\end{array}\right)\left(\begin{array}{ll}
3 & 4
\end{array}\right), \quad\left(\begin{array}{lll}
1 & 3
\end{array}\right)\left(\begin{array}{ll}
2 & 4
\end{array}\right), \quad\left(\begin{array}{lll}
1 & 4
\end{array}\right)\left(\begin{array}{ll}
2 & 3
\end{array}\right) \text {, } \\
& \underbrace{\left(\begin{array}{lll}
1 & 2
\end{array}\right)\left(\begin{array}{ll}
1 & 3
\end{array}\right)}_{\left(\begin{array}{lll}
1 & 3 & 2
\end{array}\right)} \underbrace{\left(\begin{array}{ll}
1 & 2
\end{array}\right)\left(\begin{array}{ll}
1 & 4
\end{array}\right)}_{\left(\begin{array}{lll}
1 & 4 & 2
\end{array}\right)}, \underbrace{\left(\begin{array}{lll}
1 & 2
\end{array}\right)\left(\begin{array}{ll}
2 & 4
\end{array}\right)}_{\left(\begin{array}{lll}
1 & 2 & 4
\end{array}\right)}, \underbrace{\left(\begin{array}{lll}
1 & 3
\end{array}\right)\left(\begin{array}{ll}
1 & 4
\end{array}\right)}_{\left(\begin{array}{lll}
1 & 4 & 3
\end{array}\right)}, \underbrace{\left(\begin{array}{lll}
1 & 3
\end{array}\right)\left(\begin{array}{ll}
1 & 2
\end{array}\right)}_{\left(\begin{array}{lll}
1 & 2 & 3
\end{array}\right)}, \underbrace{\left(\begin{array}{lll}
1 & 3
\end{array}\right)\left(\begin{array}{ll}
2 & 3
\end{array}\right)}_{\left(\begin{array}{lll}
2 & 3 & 1
\end{array}\right)}, \\
& \underbrace{\left(\begin{array}{lll}
1 & 4
\end{array}\right)\left(\begin{array}{ll}
1 & 3
\end{array}\right)}_{\left(\begin{array}{lll}
1 & 3 & 4
\end{array}\right)}, \underbrace{\left(\begin{array}{lll}
2 & 3
\end{array}\right)\left(\begin{array}{ll}
3 & 4
\end{array}\right)}_{\left(\begin{array}{lll}
3 & 4 & 2
\end{array}\right)}\}
\end{aligned}
$$

$\Longrightarrow A_{4}$ has 3 elements of order 2 and 8 elements of order 3 .
$D_{6}=\left\{e, r, r^{2}, r^{3}, r^{4}, r^{5}, f, f r, f r^{2}, f r^{3}, f r^{4}, f r^{5}\right\}$, on the other hand, as element $r$ with order 6. Thus $A_{4}$ and $D_{6}$ are not isomorphic.

