Rutgers University: Algebra Written Qualifying Exam

January 2017: Problem 4

Exercise. Prove that D_6 and A_4 are not isomorphic. (Here D_6e is the symmetry group of the hexagon and A_4 is the alternating group of even permutations on 4 letters.)

Solution.

$$A_4 = \{(1),\ (1\ \ 2)(3\ \ 4),\ (1\ \ 3)(2\ \ 4),\ (1\ \ 4)(2\ \ 3),\ \underbrace{(1\ \ 2)(1\ \ 3)}_{(1\ \ 3\ \ 2)}\ \underbrace{(1\ \ 2)(1\ \ 4)}_{(1\ \ 4\ \ 2)},\ \underbrace{(1\ \ 2)(2\ \ 4)}_{(1\ \ 2\ \ 4)},\ \underbrace{(1\ \ 3)(1\ \ 4)}_{(1\ \ 4\ \ 3)},\ \underbrace{(1\ \ 3)(1\ \ 2)}_{(1\ \ 2\ \ 3)},\ \underbrace{(1\ \ 3)(2\ \ 3)}_{(2\ \ 3\ \ 1)},\ \underbrace{(1\ \ 4)(1\ \ 3)}_{(1\ \ 3\ \ 4)},\ \underbrace{(2\ \ 3)(3\ \ 4)}_{(3\ \ 4\ \ 2)}\}$$

$\implies$ A_4 has 3 elements of order 2 and 8 elements of order 3.
$D_6 = \{e, r, r^2, r^3, r^4, r^5, f, fr, fr^2, fr^3, fr^4, fr^5\}$, on the other hand, as element r with order 6.
Thus A_4 and D_6 are not isomorphic.